SOME CONSIDERATIONS ON PURE ROLLING IN MULTIBODY MECHANICAL SYSTEMS

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Abstract: *The paper presents a few considerations concerning identifying the pure rolling motion in complex mechanical systems. Solving the spatial dynamic problems with dry friction is an intricate concern due to the relations which describe the friction forces from the system. Regardless the type of friction, sliding, spinning or rolling, the lack of motion is correlated by an inequality and thus the corresponding force or torque cannot be found using normal reactions. When motion occurs, a proportionality relation can be written between the force, friction torque and normal force, but the characteristic parameters of the motion remain unknown. The fact that the system behaves like a bascule bridge between two states described by different dynamic equations gives the complex character of the study. The paper presents the study of an actual spatial system and its dynamic modeling using specialized software for which a necessary criterion for identification of pure rolling is presented.*

Keywords: *rolling friction, multibody systems, dynamic simulation*

1. Introduction

The study of system dynamical behavior supposes in a first stage, obtaining the dynamical equations. a few types of parameters can be identified in the structure of dynamical equations of a system, [1]: inertial characteristics, kinematical parameters describing the position and motion of system's elements, the forces representing the interaction between the system and the external systems and the internal reactions from joints representing the constraints imposed upon relative motions between system's elements.

Considering the friction forces makes more difficult the study of system's dynamics. The complexity arises from the fact that friction forces depend on the motion of the system which, at its turn, depend on applied forces. To reduce the complexity of analyze in firs approximation, one considers the links without friction and the normal reaction forces are found, [2].

In the monographs upon the theory of mechanisms, the method of successive approximations is employed for the study of dynamic with friction, that in principle, assumes finding in a first stage, the reaction forces in the absence of friction and afterwards the kinetical-statical equations are written where the friction forces from current phase are found based on reaction forces from previous phase. The process ends when the differences between the aimed parameters, corresponding to two successive iterations are smaller than the imposed error.

The method described above proved to be very effective, [3]. One of the main limits of the method consists in the fact that the friction forces are estimated as being proportional to normal reactions. In the case of dry friction, the proportionality between friction forces or friction torques and normal reactions is valid only when relative motion occurs between contacting surfaces.

The absence of relative motion between the surfaces of a kinematical joint makes that the friction is described by inequalities which conducts to systems with unilateral systems,

[4]. Accurate modeling of system dynamics assumes writing the equilibrium equations, where friction forces and torques are defined by tribological parameters of materials from which the elements involved in the joint are made and by kinematical parameters of order zero and one (part of them unknown at this stage).

A system of second order differential equations is obtained. Integration of this system is complex due to dry friction consideration that leads to nonlinear equations of the system, [5]. Subsequently, a system with dry friction is presented, within which simultaneously act friction forces, spinning torques and rolling moments.

2. Theoretical considerations on proposed dynamical system

A system as basic as possible was intended but also with the presence of all components of friction: friction forces, friction spinning torque and rolling friction torque, [6].

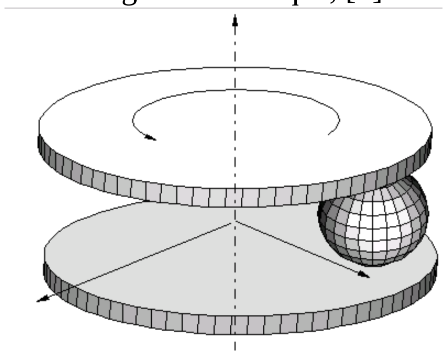

Figure 1. *Dynamic system created by two discs and a bearing ball*

The ball accomplishes two contacts with friction with each of the two discs, in $C_{1,2}$ points, Fig.1. The following forces act upon the ball, in the two points:

- Normal reactions $\boldsymbol{N}_{1,2}$ directed to the balls' centers, with direction normal to the surfaces of the two discs;
- Tangential reactions (friction forces) placed in the frontal planes of the two discs. Concerning their direction, it depends on the type of friction existing in the contact points $C_{1,2}$; for instance, if in contact points sliding occurs, the tangential reactions will have the same direction as relative velocity and will oppose to this motion. The magnitude of tangential forces are:

$$T_{1,2} = \mu_d N_{1,2} \tag{1}$$

The situation is more intricate in the case when between the contacting points there is no relative motion, state possible for immobile bodies with respect to each other or, when between the two bodies pure rolling exists. In this case, both the magnitude and the direction of the friction force are unknown. The size of friction force in this case must satisfy the condition:

$$T_{1,2} < \mu_{st} N_{1,2} \tag{2}$$

where μ_{st} represents the coefficient of static friction.

The friction forces can be characterized through the projections on two known directions. In the present case, they will be projected on radial direction giving the components $\boldsymbol{T}_{r1,2}$ and on tangential direction, (normal to radial direction), $\boldsymbol{T}_{t1,2}$

The moments action upon the ball are:

- The spinning torque:

$$\boldsymbol{M}_{sp} \leq -s_{sp1,2} N_{1,2} \frac{\omega_{sp1,2}}{|\omega_{sp1,2}|} \tag{3}$$

where $s_{sp1,2}$ is the spinning coefficient of friction and $\omega_{sp1,2}$ is the component of relative angular velocity directed by the normal to common tangent plane.

- The rolling torque:

$$\boldsymbol{M}_r \leq -s_{r1,2} N_{1,2} \frac{\omega_{r1,2}}{|\omega_{r1,2}|} \tag{4}$$

where $s_{r,1,2}$ represents the coefficient of rolling friction and $\omega_{r1,2}$ represents the rolling angular velocity, having the direction contained in the common tangent plane.

It is obvious that:

$$\omega_{1,2} = \omega_{sp1,2} + \omega_{r1,2} \tag{5}$$

In relations (3) and (4) the equality sign must be kept when the corresponding angular velocity is non-zero and the inequality sign when the velocity is lacking.

The equations describing the motion of the ball are represented by the theorem of centre of mass motion:

$$M\boldsymbol{a}_G = \boldsymbol{G} + \boldsymbol{N}_{1,2} + \boldsymbol{T}_{t_{1,2}} + \boldsymbol{T}_{r1,2} \tag{6}$$

and the moment of momentum theorem written with respect to the centre of mass G of the ball, [7]:

$$\boldsymbol{J}\varepsilon + \tilde{\omega}\boldsymbol{J}\omega = \boldsymbol{M}_{sp1,2} + \boldsymbol{M}_{r1,2} + \overline{GC}_{1,2} \times (\boldsymbol{T}_{r1,2} + \boldsymbol{T}_{t_{1,2}}) \tag{7}$$

where ω represents the angular velocity, ε the angular acceleration, $\tilde{\omega}$ the anti-symmetrical matrix associated to the angular velocity vector and $\boldsymbol{J}$ the inertia matrix calculated with respect to the centre of mass.

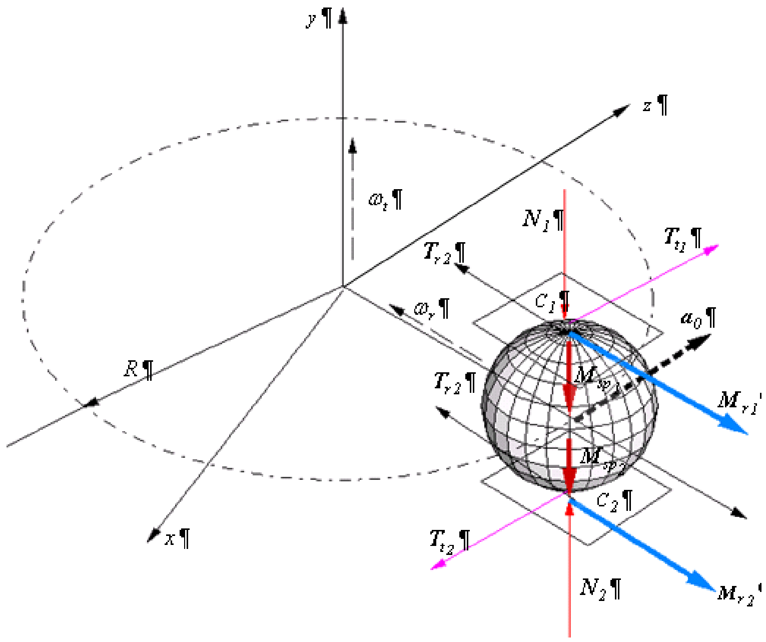

Figure 2. *Forces and torques acting upon the ball*

To the equation of motion there must be added the relations describing the friction forces and friction torques. The complexity in applying the equations of motion in an actual case appears from the fact that choosing the inequality or equality sign in the relations characteristic to friction depends on the motion status, at its turn influenced by the applied forces. Another observation is that the number of degrees of freedom of the ball depends on which of the signs of equality or inequality is kept. In the case of keeping the equality sign, the corresponding friction force or moment is known and the relative motion should be found.

In the case of keeping inequality sign, the motion can be established and the corresponding friction force or torque ought to be found. Next, the observation will be made upon friction forces. As it can be noticed, opposite to friction moments, these forces occur in both dynamical equations. For the plane-parallel motion, the situation becomes simpler because in the case of pure rolling the friction force has as unknown only the magnitude, the direction being known, and parallel to the tangent. In the spatial case, for pure rolling of the ball from Fig. 1, the friction forces present random directions. Pure rolling assumes that there is no relative motion between the points from $C_{1,2}$ contacts. It results the conclusion that the distance from the discs' axis to the contact points is constant and therefore the trajectories of centers of mass for the ball and contact points are concentrically circles.

3. Experimental set-up and modelling

To test this conclusion experimentally, on an immobile horizontal disc were placed a paper disc and an indigo paper disc, then three identical bearing balls, Fig. 3. On top of the balls a mobile disc was positioned that forms with the stationary disc a cylindrical joint. A few weights were set on the mobile disc, Fig. 4.

Figure 3 *Immobile disc with indigo paper and three identical bearing balls on top*

The superior disc (with loads) is actuated into rotation motion and afterwards let to move freely together with the entire system.

After stopping, one can notice that the balls traced circular trajectories on the paper disc, Fig. 5, fact that confirms that the centers of the balls maintained at the same distance from the discs' axis.

The circular shape of the prints is a necessary condition but not sufficient to asses that pure rolling exists between balls and discs. That because if the angular velocity should have non-zero variable component on the direction tangent to the trajectory of centre of the ball this trajectory could remain circular. In general case, the ball may effectuate three independent rotations with respect to three normal axes, attached to it. It is difficult to provide an experimental answer as all the three components should be recorded and the pure rolling condition tested.

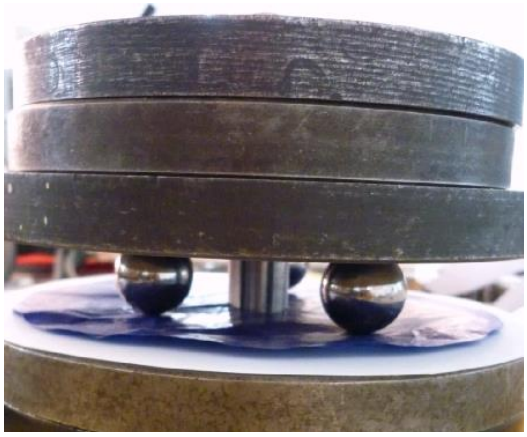

Figure 4. *Experimental set-up with mobile disc and normal loads*

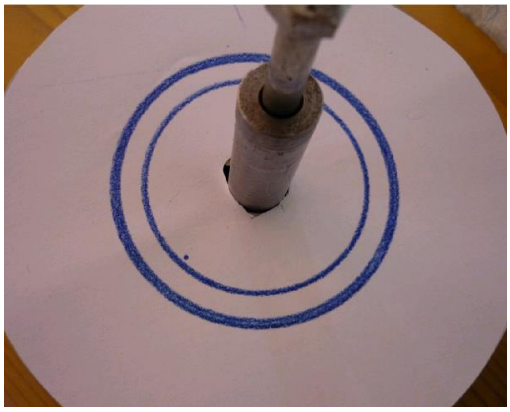

Figure 5 *Circular trajectories due to balls' motion on paper*

To give an answer referring to the existence of pure rolling, the motion from experimental set-up was modeled using dynamical simulation software, [8], Fig. 6. Two situations were considered: the motion of the upper disc is a rotation with constant angular velocity and a second case, when to the disc is applied a linearly increasing (with time) velocity.

Fig. 7 presents, for the two situations mentioned, the projections of trajectory of ball's centre on horizontal plane. It is noticed that in the first case, the trajectory is a circle. In the second case, three zones are noticed: a zone 1, where the trajectory is circular and coincides to the trajectory corresponding to the first case; a zone 2 where the centre of the ball moves away from the disc's axis (centrifugal sliding) and a zone 3, rectilinear, that in fact is the projection of parabolic trajectory of the ball in free falling.

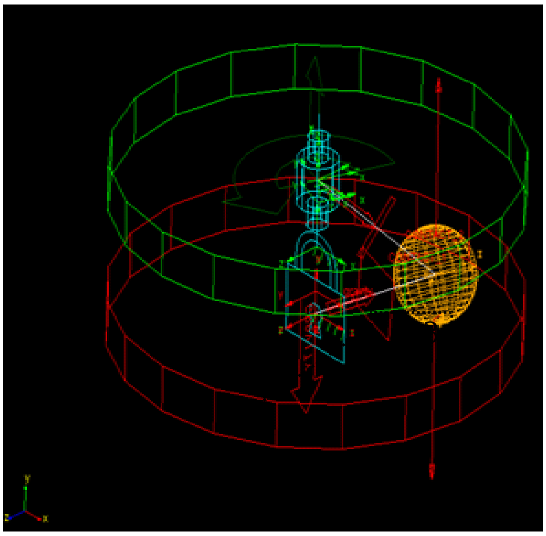

Figure 6. *Dynamical simulation of experimental set-up*

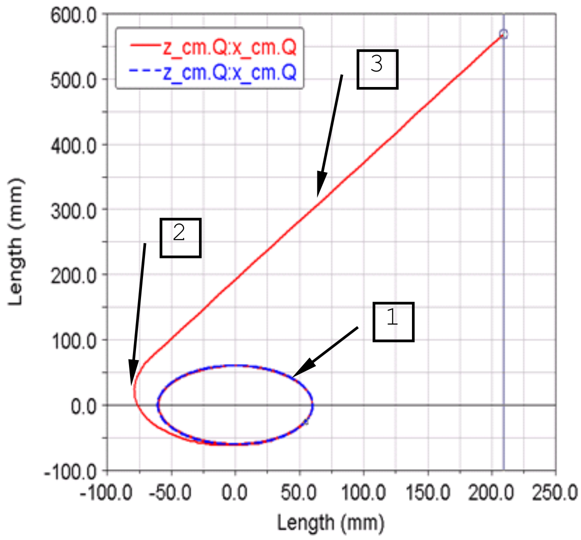

Figure 7 *Projections of trajectory of ball's centre on horizontal plane*

In Fig. 8 there are represented for the two cases, the variations of velocity magnitude of ball's centre of mass in time. In this figure also can be noticed for the second case, three zones, corresponding to pure rolling, sliding on the disc's surface and free falling.

For testing the pure rolling condition, it was represented the dependency between the dimension of angular velocity and the dimension of velocity of centre of mass. The pure rolling condition of the ball with respect to immobile disc requires that:

$$\boldsymbol{0} = \boldsymbol{v}_{C_1} = \boldsymbol{v}_G + \omega \times \boldsymbol{r}_{C_1} \qquad (8)$$

The above relation assumes proportionality between the magnitude of velocity of centre of mass and the magnitude of angular velocity. The relationship of proportionality is seen in the plot from Fig. 9 as a region where a straight line passes through origin. For both situations, the presence of this segment can be noticed.

In the first case, pure rolling exists during entire motion while for the second case, pure rolling is present only at the beginning of motion; at a certain instant the centrifugal force exceeds the sum of friction forces from radial direction and produces sliding.

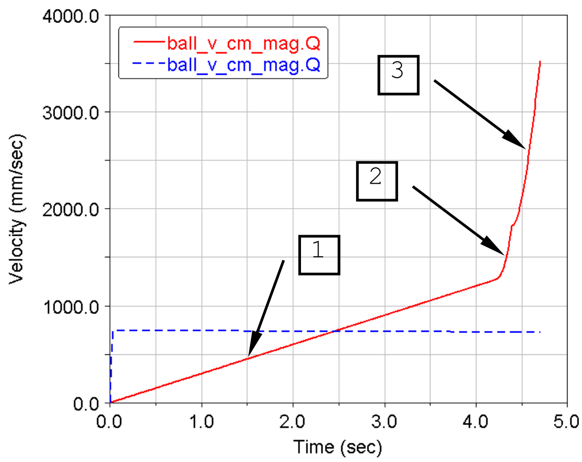

Figure 8. *Variations in time for velocity of ball's centre of mass*

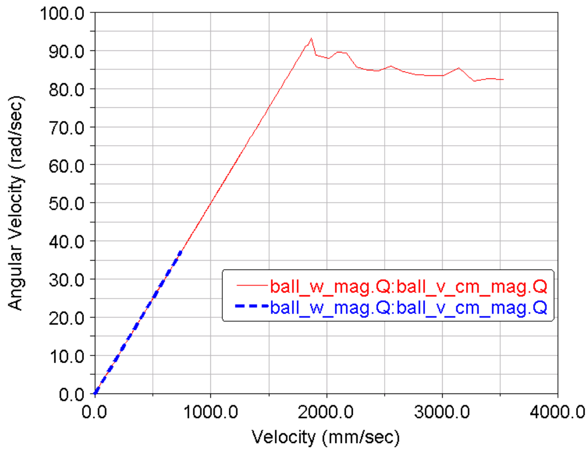

Figure 9 *Angular velocity versus velocity of centre of mass*

4. Conclusions

The paper considers a dynamic spatial system, consisting of two coaxial discs, an immobile one and the other, movable, between which a bearing ball is placed. In the proposed system there are identified all dry friction types: sliding friction, spinning friction, rolling friction, aiming to emphasize the difficulties arising in system's dynamical modeling.

Two major impediments are identified: at a given time, the friction forces depend on the motion of the system that on its turn depends on applied forces, on one side and the different manner of friction characterization in the presence or absence of relative motion from kinematical joints on the other side. For the actual system the circular trajectory of the ball's centre is confirmed. To answer to the question related to the presence of pure rolling between the discs and the ball, the system is modeled using software based on MBS method.

Considering as imposed for the mobile disc two different motion situations, it is proved that pure rolling is present for the entire motion period for one case and for the other imposed motion, three regimes are distinctive: pure rolling, sliding and free falling.

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